NONLINEAR INTERACTIONS OF ION-SOUND WAVES AND HELICONS IN A PLASMA

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The methods of perturbation theory and statistical averaging over the phases of the oscillations were used to obtain the kinetic wave equations which describe three-plasmon processes involving the merging of two ion-sound waves into a helicon and the scattering of ion-sound by plasma particles with reradiation into a helicon. The rate of accumulation of whistles in a turbulent plasma due to such nonlinear processes is estimated.

In a number of papers [1-3] dealing with turbulent heating of a plasma, data are presented which are evidence of the fact that a plasma is heated due to buildup of ion-sound instability. In [4] an attempt was made to detect ion-sound noise.

The direct observation of the latter is very difficult, and therefore in [4] measurements of whistles (helicons) which had been radiated due to nonlinear processes with the participation of ion sound were performed.

The dispersion equations for the mentioned branches of the oscillations have the form

$$\omega_s(\mathbf{k}) = \omega_{pi}[1 + (kr_{De})^{-2}]^{-1/2}$$
$$\Omega_h(\mathbf{q}) = qq_z c^2 \omega_{He} / \omega_{Pe}^2, \quad \omega_{He} \gg \Omega \gg \omega_{Hi}, \quad O_z \parallel \mathbf{H}_0 \tag{0.1}$$

Here ω, Ω are the frequencies and $\mathbf{k}$, $\mathbf{q}$ are the wave vectors of the sound and the helicons, respectively; the other notation is standard.

From Eqs. (0.1) it follows in particular that for ion-sound and helicon frequencies which are of an identical order of magnitude the latter have considerably longer wavelengths $q \ll k$; it is this which makes it possible to observe them distinct from ion-sound for which $k \gtrsim r_{De}^{-1}$. The nonlinear transformation of sound into whistles may occur via three-plasmon processes (Fig. 1a and b) and likewise via scattering of ion-sound by plasma particles with reradiation into a helicon (Fig. 1c). If the sound frequency is close to the ion plasma frequency, then the merging of two ion-sound plasmons into a helicon leads to the appearance of a narrow spectrum of whistles having a frequency close to 2 ω_{pi} in the plasma (this is shown in Fig. 2a, where $I(\Omega)$ is the intensity of the whistles).

The process shown in Fig. 1b is Cerenkov radiation of helicons by ion-sound plasmons and leads to the radiation of helicons having very long wavelengths; the frequency of these helicons does not exceed the width $\Delta\omega$ of the spectrum of the ion-sound noise (Fig. 2b). And, finally, as a consequence of the nonlinear scattering of waves by particles, helicons having $\Omega \leq \omega_s$ will be radiated (as a result of such scattering the particles of the plasma must be heated, and consequently the waves may only "redden"). Thus, the process shown in Fig. 1c yields the spectrum of helicons depicted in Fig. 2c.

The theory of nonlinear interaction of waves in a plasma situated in a magnetic field has been developed in a number of papers by various authors [5-7]. However, for the specific calculations presented in those papers general formulas are inconvenient. It is expedient to derive the kinetic equations for waves which describe the nonlinear transformation of ion-sound into helicons using the specific dispersion

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 18-25, November-December, 1970. Original article submitted January 13, 1970.

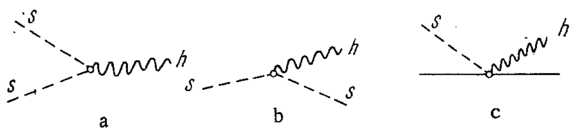

Fig. 1

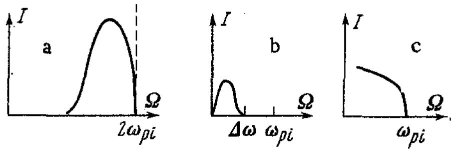

Fig. 2

properties of both oscillation modes from the very outset. The methods of calculation which shall be used have been expounded with exhaustive completeness in [8].

1. Three-Plasmon Processes with the Radiation of Helicons

The energy and momentum conservation laws allow both the merging of two ion-sound plasmons into a helicon and Cerenkov radiation of helicons by ion-sound quanta. From the experimental point of view the first of these processes is of special interest, since for a narrow spectrum of the ion-sound waves ($\omega \approx \omega_{pi}$) it, in turn, yields a clearly delineated narrow line in the spectrum. At the same time the process of Cerenkov radiation leads to the buildup of helicons having very long wavelengths, which makes their observation difficult.

Assume that in the plasma there is a high level of ion-sound noise having a frequency close to the ion plasma frequency. Since for helicons having a close frequency Ω (q) ~ ω (k) the conditions $q \ll k$ must hold, the merging process is possible only for plasmons having oppositely directed wave vectors.

We shall assume that there are such plasmons in the spectrum. We shall make use of the number of waves

$$N_{\mathbf{k}} = W_{\mathbf{k}} / \omega_k$$

as the spectral characteristic of the noise; here $W_{\mathbf{k}}$ is the spectral density of the oscillation energy. Then the kinetic equation for the wave which corresponds to the process shown in Fig. 1a has the form

$$\frac{\partial N_{\mathbf{q}}^h}{\partial t} = \sum_{\mathbf{k}} w(\mathbf{k}, \mathbf{q}) (N_{\mathbf{k}}^s N_{\mathbf{q}-\mathbf{k}}^s - N_{\mathbf{q}}^h N_{\mathbf{k}}^s - N_{\mathbf{q}}^h N_{\mathbf{q}-\mathbf{k}}^s) \tag{1.1}$$

If the level of whistles is small $W_q \ll W_k$, then their intensity increases according to a linear law:

$$H_q^2 \approx 8\pi \sum_{\mathbf{k}} w(\mathbf{k},\mathbf{q}) N_{\mathbf{k}}^s N_{\mathbf{q}-\mathbf{k}}^s \Omega t \tag{1.2}$$

It is precisely this case which is realized in the experiment described in [4]. In order to calculate the quantity w (**k**, **q**) we make use of the correspondence principle [8]; namely, we calculate w (**k**, **q**) as the coefficient of the first term in (2) on the assumption that the level of the whistles is fairly low. Then

$$\frac{\partial N_{\mathbf{q}}^h}{\partial t} = \frac{1}{\Omega} \left\langle \int dq' \mathbf{j}^{(2)}_{\mathbf{q}_\Omega} \mathbf{E}^{(2)}_{\mathbf{q}'_{\Omega'}} \right\rangle \tag{1.3}$$

$$\begin{aligned} \mathbf{j}_{\mathbf{q}}^{(2)} &= \int dk \mathbf{J}_{kq} \varphi_k \varphi_{q-k}, \quad k \equiv (\mathbf{k}, \omega) \\ \mathbf{E}_{\mathbf{q}}^{(2)} &= \int dk \mathbf{E}_{kq} \varphi_k \varphi_{q-k}, \quad q \equiv (\mathbf{q}, \Omega) \end{aligned} \tag{1.4}$$

where φ_k is the potential of the ion-sound oscillations over whose phases averaging is carried out in (1.3). All other combinations of nonlinear currents and fields either are not included in the first term of Eq. (1.1) or vanish as a result of statistical averaging over their phases.

The initial system of equations is

$$\begin{aligned} \frac{d\mathbf{v}^\alpha}{dt} &= \left(\frac{e}{m}\right)_\alpha \mathbf{E} + [\mathbf{v}^\alpha \omega_{H\alpha}] - \frac{\nabla p^\alpha}{n^\alpha m_\alpha} \\ \frac{\partial n^\alpha}{\partial t} &+ \operatorname{div}(n^\alpha \mathbf{v}^\alpha) = 0 \end{aligned} \tag{1.5}$$

$$p^{\alpha} = \text{const}\,(n^{\alpha})^{\gamma}, \quad \alpha = i,\, e, \quad \omega_{H\alpha} = \frac{e_{\alpha} H_0}{m_{\alpha} c}$$

Solving Eq. (1.5), one may neglect the ion pressure in the ion-sound oscillations, while for electrons one may place $\gamma = 1$. As far as the helicons are concerned, the thermal corrections for them are in general negligible. Further, in the indicated frequency range the ion-sound may be considered unmagnetized (i.e., one may neglect the corrections associated with the magnetic field in the shortwave terms).

In the first approximation in the amplitudes of the fields we obtain

$$\begin{gathered}
\mathbf{v}_{\mathbf{k}\omega}^{(1)e} = (kr_{De})^{-2} \frac{e}{M\omega} \mathbf{k}\varphi_k, \quad r_{De}^2 = \frac{T_e}{4\pi n e^2} \\
\mathbf{v}_{\mathbf{k}\omega}^{(1)i} = \frac{e}{M\omega} \mathbf{k}\varphi_k \\
n_{\mathbf{k}\omega}^{(1)e} = n_0 \frac{e}{M} \frac{k^2}{\omega^2} \varphi_k (kr_{De})^{-2} \\
n_{\mathbf{k}\omega}^{(1)i} = n_0 \frac{e}{M} \frac{k^2}{\omega^2} \varphi_k, \quad M \equiv m_i
\end{gathered} \tag{1.6}$$

In the next approximation in the amplitudes of the fields the system (1.5) yields

$$\mathbf{v}_q^{(2)i} = -\frac{1}{2}\left(\frac{e}{M}\right)^2 \int dk\, \frac{k^2 \mathbf{q}}{\Omega\omega(\Omega-\omega)} \left(\varphi_k \varphi_{q-k} - \langle \varphi_k \varphi_{q-k} \rangle\right) \tag{1.7}$$

$$\mathbf{v}_q^{(2)e} = \frac{1}{2}(kr_{De})^{-4}\left(\frac{e}{M}\right)^2 \int dk\, \frac{k^2}{\omega(\Omega-\omega)} \left\{\frac{(\mathbf{q}\boldsymbol{\omega}_{He})\,\boldsymbol{\omega}_{He}}{\Omega\omega_{He}^2} - i\frac{[\boldsymbol{\omega}_{He}\mathbf{q}]}{\omega_{He}^2}\right\} \left(\varphi_k \varphi_{q-k} - \langle \varphi_k \varphi_{q-k} \rangle\right) \tag{1.8}$$

In (1.7), (1.8) we retain only those terms having field combinations which are included in the first term of Eq. (1.1).

From (1.6)-(1.8) we can obtain the required nonlinear current:

$$\mathbf{j}_q^{(2)} = \sum_{\alpha=i,\,e} e_{\alpha}\left(n_0 \mathbf{v}_q^{(2)\alpha} + \int dk\, n_k^{(1)\alpha} \mathbf{v}_{q-k}^{(1)\alpha}\right)$$

With an accuracy of up to terms of order ω/ω_{He}, $(kr_{De})^{-4}$ we have

$$\mathbf{j}_q^{(2)} = \mathbf{j}_q^{(2)i} = \frac{n_0 e}{2}\left(\frac{e}{M}\right)^2 \int dk\, \frac{k^2}{\omega(\Omega-\omega)} \left(\varphi_k \varphi_{q-k} - \langle \varphi_k \varphi_{q-k} \rangle\right) \left\{\left[\frac{1}{\Omega-\omega} - \frac{1}{\omega}\right]\mathbf{k} + \left[\frac{1}{\omega} - \frac{1}{\Omega}\right]\mathbf{q} - 2\frac{\mathbf{kq}}{k^2}\frac{\mathbf{k}}{\Omega-\omega}\right\} \tag{1.9}$$

Further, the nonlinear currents and fields are interrelated by the equation

$$\left(q^2\delta_{ij} - q_j q_i - \frac{\Omega^2}{c^2}\varepsilon_{ij}\right) E_{qj}^{(2)} = \frac{4\pi i\Omega}{c^2} j_{qi}^{(2)} \tag{1.10}$$

where

$$\varepsilon_{ij} = \begin{Vmatrix} \varepsilon_0 & ig & 0 \\ -ig & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_1 \end{Vmatrix} O_z \parallel \mathbf{H}_0$$

$$\varepsilon_0 = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega^2 - \omega_{H\alpha}^2}, \quad g = -\sum_{\alpha} \frac{\omega_{H\alpha}\omega_{p\alpha}^2}{\Omega(\Omega^2 - \omega_{H\alpha}^2)}, \quad \varepsilon_1 = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega^2}$$

As a result of simple algebraic transformations one can obtain

$$\mathbf{E}_q^{(2)} = \Lambda(q, \Omega)\, \mathbf{j}_q^{(2)} \tag{1.11}$$

from (1.10).

Here the tensor Λ_{ij} has the following form with an accuracy of up to terms $\sim \Omega/\omega_{He}$:

$$\Lambda_{ij} = i\frac{4\pi\Omega^3}{q^2 q_z^2 c^2} (\Omega_q^2 - \Omega^2)^{-1} Q_{ij} \approx 2\pi^2 \frac{\Omega_q^2}{q^2 q_z^2 c^2} \delta(\Omega - \Omega_q)\, Q_{ij} \tag{1.12}$$

The components of the tensor Q_{ij}, where i, j = 1, 2, 3, are determined by the following expressions:

$$Q_{11} = q_x^2 + q_z^2, \quad Q_{12} = q_x q_y + iqq_z, \quad Q_{21} = q_x q_y - iqq_z, \quad Q_{22} = q_y^2 + q_z^2$$
$$Q_{3i} = Q_{j3} = 0$$

Substituting (1.12) into (1.3), (1.4) and carrying out statistical averaging over the phases of the waves, we obtain

$$\frac{\partial N_q^h}{\partial t} = \frac{1}{\Omega} \int dk \mathbf{J}_{k,q} \Lambda \mathbf{J}_{-k,-q} |\varphi|_k^2 |\varphi|_{q-k}^2$$

Then, finally, going over from Fourier components in the space (**k**, ω; **q**, Ω) to Fourier components with respect to **k** and **q**, we obtain an equation of the type (2) after simple transformations, where the matrix element has the form

$$w(\mathbf{k}, \mathbf{q}) = \frac{\pi}{2} \frac{\omega_{pi}^6 \Omega}{\omega(\Omega - \omega)} \frac{\delta(\Omega_q - \omega_k - \omega_{q-k})}{n_0 M c^2} q^{-2} q_z^{-2} \mathbf{A} \Lambda_0 \mathbf{A}$$
$$A = \left(\frac{1}{\Omega - \omega} - \frac{1}{\omega}\right)\mathbf{k} + \left(\frac{1}{\omega} - \frac{1}{\Omega}\right)\mathbf{q} - 2\frac{\mathbf{kq}}{k^2}\frac{\mathbf{k}}{\Omega - \omega}, \quad \Lambda_{0ij} = \operatorname{Re} Q_{ij} \tag{1.13}$$

Equation (1.13) acquires a very simple form for the case of narrow ion-sound spectra when $\Omega \approx 2\omega \approx 2\omega_{pi}$. In this case

$$w(\mathbf{k}, \mathbf{q}) = \frac{\pi \omega_{pi}^3}{q^2 q_z^2} \frac{\delta(\Omega_q - \omega_k - \omega_{q-k})}{n_0 M c^2} \left[\frac{1}{4} q^2 q_\perp^2 - 2\frac{\mathbf{kq}}{k^2} q^2 (\mathbf{k}_\perp \mathbf{q}_\perp) + 4\left(\frac{\mathbf{kq}}{k^2}\right)^2 (q_z^2 k_\perp^2 + (\mathbf{k}_\perp \mathbf{q}_\perp)^2)\right]$$
$$q_\perp = q - \frac{(\mathbf{qH}_0)\mathbf{H}_0}{H_0^2}, \quad \mathbf{k}_\perp = \mathbf{k} - \frac{(\mathbf{kH}_0)\mathbf{H}_0}{H_0^2} \tag{1.14}$$

Under the conditions of the experiment described in [4] (i.e., for a small initial level of whistles) the growth of the intensity of the whistles takes place according to the following law on the basis of (1.2):

$$H_\sim^2 \sim \frac{(W^s)^2}{n_0 M c^2} \omega_{pi} t, \quad W^s = \int d\mathbf{k} W_\mathbf{k} \tag{1.15}$$

This result coincides with the estimate made in [4] and is in good agreement with experimental data. Under these conditions the radiation spectrum corresponds to Fig. 2a.

2. The Scattering of Ion Sound by Particles with Reradiation into Helicons

The process of ion-sound scattering by particles with reradiation into whistles (the diagram shown in Fig. 1c) may be described by the following kinetic equation:

$$\frac{\partial N_\mathbf{q}^h}{\partial t} = N_\mathbf{q}^h \sum w(\mathbf{k}, \mathbf{q}) N_\mathbf{k}^s \equiv 2\gamma_H(\mathbf{q}) N_\mathbf{q}^h \tag{2.1}$$

It is easy to see that unlike Eq. (1.1) an equation of the type (2.1) has only exponential solutions (i.e., if pumpover of ion sound into the helicons can actually occur effectively via scattering of particles, then an exponentially rapid growth of the level of the whistles must be observed experimentally). For the process displayed in Fig. 1c the structure of the particle distribution function is essential; therefore, instead of the system (1.5) it is necessary to solve the system of equations developed by A. A. Vlasov using perturbation theory. In order to reduce the excessively cumbersome calculations we shall solve a model problem: nonlinear interactions of waves belonging to two one-dimensional spectra (i.e., we place $\mathbf{q} \parallel \mathbf{k} \parallel \mathbf{H}_0$). This allows us at the same time to eliminate three-plasmon processes from consideration; the probability of these processes, as can easily be seen from (1.14), vanishes in this case. Henceforth we shall indicate the extent to which the result may change when the transition is made from one-dimensional spectra to three-dimensional ones. As the original system of equations we shall take the Maxwell equations and the kinetic equations

$$\frac{\partial f^\alpha}{\partial t} + \mathbf{v}\nabla f^\alpha + \left(\frac{e}{m}\right)_\alpha \left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right) \frac{\partial f^\alpha}{\partial \mathbf{v}} = 0, \quad \alpha = i, e \tag{2.2}$$

In order to calculate the probability w (**k**, **q**) which is incorporated in Eq. (2.1) it is necessary to obtain the nonlinear current

$$j_q^{(3)} = \sum_{\alpha=i,e} ne_\alpha \int f_q^{(3)\alpha} \mathbf{v}\, d\mathbf{v}$$

which is proportional to a combination of fields of the type $\varphi_k \varphi_{k'} E_q$.

In the first approximation the solution of Eq. (2.2) is

$$f_k^{\alpha(1)} = -\left(\frac{e}{m}\right)_\alpha \frac{\mathbf{k}\partial f^\alpha / \partial \mathbf{v}}{\omega - \mathbf{kv} + i\varepsilon} \varphi_k, \quad \varepsilon > 0$$

$$f_q^{i(1)} = -i\left(\frac{e}{M}\right) \frac{\mathbf{E}_q \partial f^i / \partial \mathbf{v}}{\Omega - \mathbf{qv}}$$

$$f_q^{e(1)} = \frac{e}{m\omega_{H_e}} \exp \frac{i(\Omega - \mathbf{qv})\varphi}{\omega_{H_e}} \left\{ \int_0^\varphi \exp \frac{-i(\Omega - \mathbf{qv})\varphi'}{\omega_{H_e}} \frac{\partial f^e}{\partial \mathbf{v}} d\varphi' + \mathbf{C} \right\} \mathbf{E}_q \tag{2.3}$$

$$\mathbf{C} = \left(\exp \frac{-2\pi i(\Omega - \mathbf{qv})}{\omega_{H_e}} - 1\right)^{-1} \int_0^{2\pi} \exp \frac{-i(\Omega - \mathbf{qv})\varphi'}{\omega_{H_e}} \frac{\partial f^e}{\partial \mathbf{v}} d\varphi'$$

Using simple transformations (see, for example, [9]), the expression for a longwave electronic perturbation may be represented in the form of a series whose principal term takes the following form with allowance for the smallness of the ratio Ω/ω_{He}:

$$f_q^{e(1)} = \frac{2}{\omega_{H_e} v_{T_e}^2} \frac{e}{m} f^e \frac{\mathbf{H}_0}{H_0} \mathbf{v}\mathbf{E}_q, \quad v_{T_e}^2 = \frac{2T_e}{m} \tag{2.4}$$

A correction of the same order of magnitude appears in the nonunidimensional spectrum. Here and further on we assume that the unperturbed distribution functions f^α are Maxwellian – this allows us to represent the results in finite form.

Let us write out the equation of the third approximation in the amplitude of the field for longwave perturbations:

$$\begin{aligned} &-i(\Omega - \mathbf{qv}) f_q^{\alpha(3)} - \omega_{H_\alpha} \partial f_q^{\alpha(3)} / \partial \varphi \\ &= -\left(\frac{e}{m}\right)_\alpha \int dk' \left(\mathbf{E}_{k'} + \frac{\mathbf{v}}{\omega'} \times \mathbf{k}' \times \mathbf{E}_{k'}\right) \frac{\partial}{\partial \mathbf{v}} f_{q-k'}^{\alpha(2)} \\ &-\left(\frac{e}{m}\right)_\alpha \int dk' \frac{\partial}{\partial \mathbf{v}} f_{k'}^{\alpha(1)} \left(\mathbf{E}_{q-k'}^{(2)} + \frac{\mathbf{v}}{\Omega - \omega'} \times (\mathbf{q} - \mathbf{k}') \times \mathbf{E}_{q-k'}^{(2)}\right) \end{aligned} \tag{2.5}$$

The vectors $\mathbf{k}'$ and $\mathbf{q}-\mathbf{k}'$ in the right side of (2.5) simultaneously belong to the longwave (helicon) or shortwave (ion-sound) regions of the spectrum. In calculating the term which is proportional to the combination of fields $\varphi_k \varphi_{k'} E_{q'}$, one should retain summation only over the shortwave region in the right side of (2.5); all other terms containing such a combination of fields vanish as a result of averaging over the phases. As a result, Eq. (2.5) is substantially simplified:

$$-i(\Omega - \mathbf{qv}) f_q^{\alpha(3)} - \omega_{H_\alpha} \partial f_q^{\alpha(3)} / \partial \varphi = i\left(\frac{e}{m}\right)_\alpha \frac{\partial}{\partial \mathbf{v}} \int dk \left[\mathbf{k}\varphi_k f_{q-k}^{\alpha(2)} + (\mathbf{q} - \mathbf{k}) f_k^{\alpha(1)} \varphi_{q-k}^{(2)}\right] \tag{2.6}$$

Thus, in the second approximation it is necessary to calculate only the longwave perturbations $f_k^{\alpha(2)}$.

The result of the calculations has the form

$$\begin{aligned} f_k^{\alpha(2)} = &\, i\left(\frac{e}{m}\right)_\alpha^2 \int dq \varphi_{k-q} \frac{1}{\omega - \mathbf{kv} + i\varepsilon} \left[\left(1 + \frac{\mathbf{qv}}{\Omega}\right)\mathbf{E}_q - \frac{(\mathbf{E}_q \mathbf{v})}{\Omega} \mathbf{q}\right] \frac{\partial f^\alpha / \partial \mathbf{v}}{\omega - \Omega - \mathbf{kv} + i\varepsilon} \\ &- \left(\frac{e}{m}\right)_\alpha \int dq \varphi_{k-q} \frac{\mathbf{k}\partial f^{\alpha(1)} / \partial \mathbf{v}}{\omega - \mathbf{kv} + i\varepsilon} \end{aligned} \tag{2.7}$$

In Eq. (2.7) terms which do not contain the required combinations of fields are omitted.

It may be shown that the nonlinear potential $\varphi_k^{(2)}$ in the one-dimensional model vanishes in accordance with the results of the preceding section which were obtained in the hydrodynamic approximation.

Let us first obtain the kinetic equations for the waves which describe scattering by ions. In the second approximation we have

$$f_k^{i(2)} = \int dq \mathbf{V}_{k,q} \mathbf{E}_q \varphi_{k-q} \tag{2.8}$$

$$\mathbf{V}_{k,q} = i\left(\frac{e}{M}\right)^2 \left[\frac{\partial / \partial \mathbf{v}}{\omega - \mathbf{kv} + i\varepsilon} \frac{\mathbf{k}\partial f^i / \partial \mathbf{v}}{\omega - \Omega - \mathbf{kv} + i\varepsilon} + \frac{\mathbf{k}\partial / \partial \mathbf{v}}{\omega - \mathbf{kv} + i\varepsilon} \frac{\partial f^i / \partial \mathbf{v}}{\Omega - \mathbf{qv}}\right] \tag{2.9}$$

From (2.6), (2.8), (2.9) one can easily obtain

$$f_q^{i(3)} = -\frac{e}{M}\int dk dq' \frac{\mathbf{k}\partial / \partial \mathbf{v}}{\Omega - \mathbf{qv}} \mathbf{V}_{q-k,q'} \mathbf{E}_{q'} \varphi_k \varphi_{q-k-q'}$$

Henceforth in averaging over the phases of the ion-sound oscillations the integration with respect to dq' is removed because of the δ-function $\delta(\mathbf{q}-\mathbf{q}')\,\delta(\Omega-\Omega')$. We shall assume that this operation has been performed. For the nonlinear current $j^{(3)}$ we obtain the following expression:

$$\mathbf{j}_q^{(3)i} = \int n e f_q^{(3)i} \mathbf{v} d\mathbf{v} = \frac{4\pi n e}{v_{T_i}^4}\left(\frac{e}{M}\right)^3 \int dk\, |\varphi|_k^2 \int d\mathbf{v}\mathbf{v} \frac{\mathbf{k}\partial / \partial \mathbf{v}}{\Omega - \mathbf{qv}} \frac{(\mathbf{E}_q \mathbf{v}) f^i}{\omega - \Omega - \mathbf{kv} - i\varepsilon} \left\{\frac{\mathbf{kv}}{\omega - \mathbf{kv} - i\varepsilon} + \frac{1}{2}\frac{kqv_{T_i}^2}{(\Omega - \mathbf{qv})^2} - \frac{\mathbf{kv}}{\Omega - \mathbf{qv}}\right\}, \tag{2.10}$$

$$v_{T_i}^2 = \frac{2T_i}{M}$$

We substitute Eq. (2.6) into the equation

$$\partial W / \partial t = \mathbf{j}^{(3)}\mathbf{E}$$

and perform averaging over the phases of the longwave oscillations. Going over to the Fourier components with respect to **k**, **q**, we obtain

$$\frac{\partial W_\mathbf{q}}{\partial t} = \int d\mathbf{k}\, |\varphi|_\mathbf{k}^2 \Phi_{kq}$$

where

$$\Phi_{kq} = -i\frac{\omega_{pi}^2}{v_{T_i}^4}\left(\frac{e}{M}\right)^2 \int d\mathbf{v} \frac{kq}{(\Omega - \mathbf{qv})^2} \left\{\frac{(\mathbf{E_q v})(\mathbf{E_q^+ v})(\mathbf{kv}) f^i}{(\omega - \Omega - \mathbf{kv} - i\varepsilon)(\omega - \mathbf{kv} - i\varepsilon)} + \frac{kqv_{T_i}^2}{2}\frac{(\mathbf{E_q v})(\mathbf{E_q^+ v}) f^i}{(\omega - \Omega - \mathbf{kv} - i\varepsilon)(\Omega - \mathbf{qv})^2}\right. \tag{2.11}$$

$$\left. - \frac{(\mathbf{kv})(\mathbf{E_q v})(\mathbf{E_q^+ v}) f^i}{(\omega - \Omega - \mathbf{kv} - i\varepsilon)(\Omega - \mathbf{qv})}\right\} \tag{2.12}$$

Only the real part of Φ_{kq} and consequently the imaginary part of the integrals included therein are essential for the evolution of the spectrum. However, the imaginary contribution from the bypassing of poles of the type $(\omega - \mathbf{kv})^{-1}$, $(\Omega - \mathbf{qV})^{-1}$ is exponentially small. This allows simplification of the expression for Φ_{kq}:

$$\mathrm{Re}\,\Phi_{kq} = -\frac{\omega_{pi}^2}{\pi v_{T_i}^4}\left(\frac{e}{M}\right)^2 \frac{kq}{\Omega^2}\left(\frac{\omega - \Omega}{\omega} - \frac{\omega - \Omega}{\Omega} + \frac{kqv_{T_i}^2}{2\Omega^2}\right) \mathrm{Im} \int d\mathbf{v} \frac{(\mathbf{E_q v})(\mathbf{E_q^+ v}) f^i}{\omega - \Omega - \mathbf{kv} - i\varepsilon} \tag{2.12}$$

The integral which is included in this equation can be reduced to Cramp functions and acquires an especially simple form for a small frequency difference

$$\frac{\omega - \Omega}{kv_{T_i}} \ll 1$$

As a result we obtain an equation of the type (2.1):

$$\frac{\partial N_\mathbf{q}}{\partial t} = 2\gamma_H^i(\mathbf{q}) N_\mathbf{q}$$

$$\gamma_H^{(i)}(\mathbf{q}) = \frac{2\sqrt{\pi}}{3}\omega_{pi}\int \frac{d\mathbf{k} N_\mathbf{k} \omega_\mathbf{k}}{nMc^2}\frac{k}{q}\left(\frac{\omega_{pi}}{kv_{T_i}}\right)^3 \frac{(\omega - \Omega)^2}{\omega\Omega} \tag{2.13}$$

Calculations are carried out analogously when scattering by electrons is considered. We shall not present these calculations in view of their still more cumbersome nature, but we shall present the result immediately: the estimate for a nonlinear growth rate $\gamma_H^{(e)}$ (q) describing scattering with reradiation via electrons is

$$\gamma_H^{(e)}(\mathbf{q}) \approx 2\sqrt{\pi}\omega_{pe}\int \frac{d_{\mathbf{k}} N_{\mathbf{k}}\omega_k}{nMc^2}\left(\frac{\omega_{pe}}{kv_{T_e}}\right)^3\left(\frac{\Omega}{\omega_{H_e}}\right)^2\frac{k}{q} \tag{2.14}$$

Thus,

$$\gamma_H^{(e)} \sim \gamma_H^{(i)}\sqrt{\frac{M}{m}}\left(\frac{\Omega}{\omega_{H_e}}\right)^2\left(\frac{T_i}{T_e}\right)^3 \tag{2.15}$$

i.e., under typical experimental conditions (in particular, in the experiment described in [4] also) scattering by ions introduces a more substantial contribution to the evolution of the spectrum.

All of the calculations carried out in the present section were carried out within the framework of the one-dimensional model $\mathbf{k}\|\mathbf{q}\|\mathbf{H}_0$. In three-dimensional spectra $\gamma_H^{(i)}$ may vary by an amount of the same order.

As far as the quantity $\gamma_H^{(e)}$ is concerned, it follows that in solving Eq. (2.6) in the non-one-dimensional spectrum the results may be larger by a factor of ω_{H_e}/Ω. The ratio $\gamma^{(e)}/\gamma^{(i)}$ remains small in this case also.

In [4] the integral density of ion-sound noise, as measured from the intensity of the radiation of helicons due to three-plasmon processes, turned out to be of the order of 10^{-2} erg/cm^3. For the plasma parameters $n \sim 10^{13}$ cm^{-3}, $T_e \sim 100$ eV one may calculate

$$\gamma_H^{(i)} \leqslant \omega_{pi}\cdot 10^{-6}$$

from (2.13).

For a recording time $\tau \sim 0.1\,\mu$sec we obtain $\gamma_H^{(i)}\tau < 10^{-2}$; i.e., the helicons radiated due to scattering of ion sound by particles cannot be recorded. In fact, no exponential growth of the energy density of the helicons was observed in the experiment described in [4]. At the same time a clearly defined intensity peak of the longwave noise was recorded for $\Omega \sim 2\omega_{pi}$, the energy density being of an order of magnitude corresponding to Eq. (1.15).

The author thanks L. I. Rudakov for supervising the work.

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